2023

Vietnam Northern Provincial Programming Contest

University of Engineering and Technology
Faculty of Information Technology
8/10/2023

A. FIB

The Fibonacci sequence is defined as follows: $F_0 = 1$, $F_1 = 1$, $F_n = F_{n-2} + F_{n-1}$. Some of the first terms of the Fibonacci sequence: 1; 1; 2; 3; 5; 8; 13; 21; 34; ...

Given a natural number n, count the number of ways to express it as a product of Fibonacci numbers (numbers in the expression must be greater than 1).

INPUT

The first line contains the number of tests ($1 \le t \le 50$). Each of the next t lines contains an integer n ($2 \le n \le 10^{18}$).

OUTPUT

For each test case, print on one line the number of ways to factor n into the product of Fibonacci numbers.

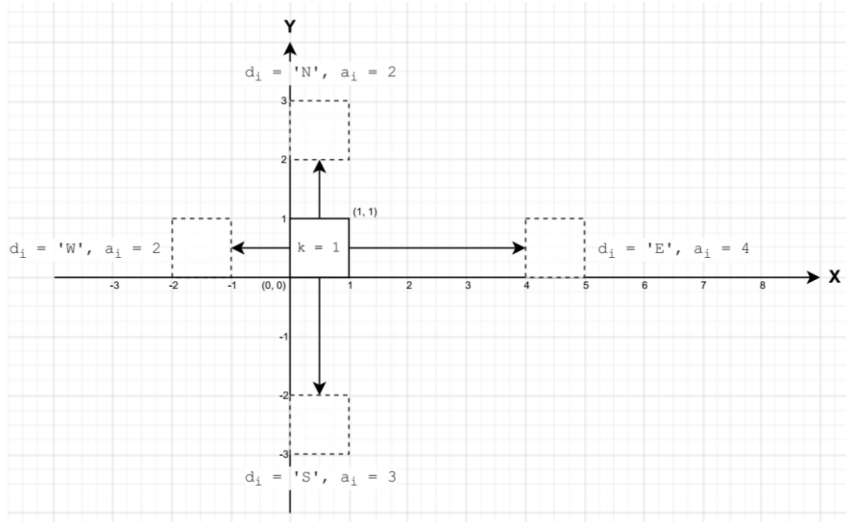
Sample Input	Sample Output
5 2 7 8 40 64	1 0 2 2 3

Example explanation:

- The number 2 can be expressed as a product of Fibonacci numbers uniquely as 2 = 2
- The number 7 cannot be represented as a product of Fibonacci numbers
- The number 8 can be represented in two ways: 8 = 2 × 2 × 2 and 8 = 8
- The number 40 can be represented in two ways: 40 = 2 × 2 × 2 × 5 and 40 = 5 × 8

A man is sitting at home when he hears someone knocking at the door
Knock
Knock
Knock knock
Knock knock knock
Knock knock knock knock knock
"Who's there?"
"Fibonacci"

B. ROBOT



A coordinate plane is planned to be cleaned using a robot cleaner. The robot has a square shape of size k×k, the edges are parallel to the coordinate axis. Initially, the lower left corner of the robot is at point (0; 0), the upper right corner is at point (k; k). Given a series of n movements on the plane, the i-th movement is characterized by the direction of movement, taking the values 'N' (up, increasing Y coordinate), 'S' (down, decreasing Y coordinate) , 'W' (left, decreasing X coordinate) or 'E' (right, increasing X coordinate) and integer a_i - the distance the robot moves.

The robot cleans the entire area underneath it at any time. In other words, a point is considered clean if and only if it belongs to a square of size k × k in which the robot occupies at some time. Based on the given movements of the robot, calculate the total area of the entire cleaned surface.

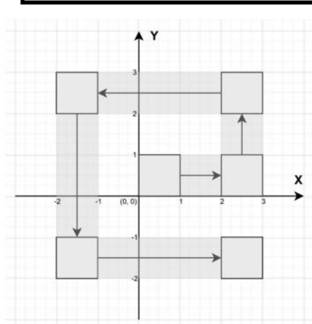
INPUT

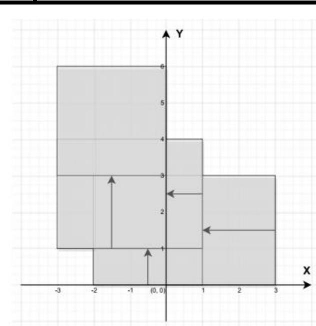
The first line contains two integers: the size of robot k and the number of movements n ($1 \le k \le 10^4$; $1 \le n \le 10^5$). The i-th line in the next n lines contains the direction of movement d_i and the distance of movement a_i (d_i - are the letters 'N', 'S', 'W' or 'E'; $1 \le a_i \le 10^9$)

OUTPUT

Print the total surface area cleaned by the robot.

Sample Input	Sample Output
1 5 E 2 N 2 W 4 S 4 E 4	17
3 4 W 2 N 1 W 1 N 2	27





C. PERMUTATION

Given a set A containing some number of integers ranging from 1 to 8. Consider the sequence consisting of n integers [a_1; a_2; …; a_n], each chosen from set A.

We will call this sequence beautiful if for any number x, the elements of the sequence equal to x are at least x apart. That is, for any number x and for two indices $1 \le i < j \le n$, such that $a_i = a_j = x$, we always have $j - i \ge x$.

Count the number of beautiful sequences with the given number n and set A. Because this number is very large, calculate it modulo $10^9 + 7$.

INPUT

The first line contains two integers n and m - the length of the sequence and the number of elements of set A ($1 \le n \le 100$, $1 \le m \le 8$). The second line contains m different integers a_i in ascending order - elements of set A ($1 \le a_i \le 8$, $a_i < a_{i+1}$)

OUTPUT

Print the answer, modulo $10^9 + 7$.

Sample Input	Sample Output
3 2 1 2	5

Example explanation: The sequences are [1; 1; 1], [1; 1; 2], [1; 2; 1], [2; 1; 1], [2; 1; 2].

D. JUSTCOUNT

You are given a grid with n rows and m columns. Each cell of the grid can be colored black or white.

A grid is beautiful if exactly k cells are colored black, and no two black cells share a corner or an edge.

Count the number of beautiful grids, modulo 123456789.

INPUT

The first and only line contains three positive integers n, m, and k. ($1 \le k \le n \times m \le 256$)

OUTPUT

Print the answer, modulo 123456789.

Sample Input	Sample Output
2 2 1	4

E. PIZZA

Alex has a pizza that he wants to share with his friends. The pizza is in the shape of a convex polygon with N sides. He decides to make cuts along the diagonals of the polygon.

After all the cuts have been made, count the number of pizza slices there are in total. As the answer may be large, please print it modulo 987654321.

Note that it is guaranteed that no three diagonals of the polygon intersect at a point.

INPUT

The first and only line contains the positive integer N. ($3 \le N \le 10^{18}$)

OUTPUT

Print the answer, modulo 987654321.

Sample Input	Sample Output
4	4

F. BUREAUCRACY

There is a node-weighted, root-fixed tree of N vertices with indices running from 0 to N-1, and vertex 0 is the root. You are given Q queries of three types:

- 1 u w. First the edges connecting u and its children are deleted. Then, a new vertex indexed with the first unused index (that is, the first vertex created from queries of this type will have index N, second will have N+1, etc.) is created, with w as its weight, u as its parent and u's former children as its children.
- 2 u, vertex u is deleted, and its children will become children of u's parent. It is guaranteed that u is not the root.
- 3 u v k, you are asked to output the k-th smallest weight on the path connecting u and v. k is guaranteed to be no greater than the number of nodes on the path connecting u and v (i.e. $k \le d(u, v) + 1$ with $d(u, v)$ being the distance between u and v)

INPUT

The first line contains 2 integers N and Q. ($N, Q \le 5\times10^4$)

The second line contains N integers, the i-th of which denotes the weight of vertex i-1. All weights are between 0 and 10^9, inclusive.

Each of the next N-1 lines contains 2 integers u and v, denoting there is an edge between u and v.

Finally, Q lines follow, describing queries in the format above.

OUTPUT

For each of query of type 3, output the answer on a new line.

Sample Input	Sample Output
10 10 364715055 598838324 844502191 211147053 438769309 303905477 513518273 332723869 92018216 34702917 1 5 3 2 2 5 6 8 7 5 5 8 8 4 4 0 0 9 3 2 0 1 1 4 951058488 3 9 2 1 1 9 847945860 2 11 1 4 873328923 2 8 1 4 743926561 2 4 1 0 390537674	92018216 34702917

G. SUBSEQUENCE

You are given a string S with length N and a positive integer K. Count the number of distinct strings that can be obtained by removing exactly K characters from the string. Since the answer may be large, print it modulo $10^9 + 7$.

INPUT

The first line contains two integers N and K - the length of the string S and the number of characters to be removed. ($2 \leq N \leq 2 \times 10^5$, $1 \leq K \leq \min(10, N - 1)$)

The second line contains the string S with length N. All characters in the string are lowercase English letters.

OUTPUT

Print the answer, modulo $10^9 + 7$.

Sample Input	Sample Output
9 2 aaabbbccc	6

H. INVERSIONS

A permutation of length N is an array containing each integer from 1 to N exactly once. An inversion of a permutation P is a pair of positions (i, j) such that $i < j$ and $P_i > P_j$.

You are given three integers N, M, and K. Print the K-th lexicographically smallest permutation of length N that has exactly M inversions. If there is no such permutation, print -1 instead.

INPUT

The first and only line contains three integers N, M, and K ($1 \le N \le 100$, $0 \le M \le N \times (N - 1) / 2$, $1 \le K \le 10^{18}$)

OUTPUT

Print one line containing N integers - the required permutation. If there is no such permutation, print -1 instead.

Sample Input	Sample Output
5 1 2	1 2 4 3 5
5 0 2	-1

I. NIM

Alice and Bob are going to play a variation of Nim.

To set up the game, first Bob chooses a strictly increasing sequence of N integers: $A_1, A_2, A_3, \ldots, A_N$. Then, Alice arranges K piles of stones in a row. The number of stones in each pile must be a number in Bob's sequence. All piles are distinct from each other, so there is a total of N^K different starting configurations.

Alice and Bob take turns making moves, with Alice going first. For each move, a player must choose 1 or 2 piles and remove a positive number of stones from each of the chosen piles. It is allowed to remove different amounts of stones from each pile. The player who cannot make a valid move loses.

Assuming Alice and Bob play optimally, count the number of different starting configurations that result in Alice losing and Bob winning. Since the answer may be large, print it modulo $10^9 + 9$.

INPUT

The first line contains two positive integers N and K - the length of Bob's sequence, and the number of piles of stones. ($1 \le N \le 1000$, $1 \le K \le 10^{18}$)

The second line contains N integers $A_1, A_2, A_3, \ldots, A_N$ - the numbers in Bob's sequence. ($1 \le A_1 < A_2 < \ldots < A_N \le 1000$)

OUTPUT

Print the answer, modulo $10^9 + 9$.

Sample Input	Sample Output
2 3 1 2	2
9 123456789 1 2 3 4 5 6 7 8 9	552110917

J. KAMI

KAMI (Kami's Amazing Mathematical Instructions) is a simple programming language.

There are 676 variables that can be used, which are represented by pairs of English letters: AA, AB, AC, ..., ZX, ZY, ZZ. All variables are unsigned 64-bit integers. That is, their values range from 0 to 2^{64} - 1.

There are three types of instructions:

- ADD A B C - Assign the value of the sum of variable A and variable B (modulo 2^{64}) to variable C.
- MUL A B C - Assign the value of the product of variable A and variable B (modulo 2^{64}) to variable C.
- MOD A B - Assign the value of variable A modulo P to variable C, where P is some fixed prime number.

Your task is to write a program in this language that can compute the value of the N-th Fibonacci number modulo P for all integer values of N that satisfy $1 \leq N \leq 2^{64} - 1$.

The Fibonacci sequence is defined as follows: $F_0 = 1$, $F_1 = 1$, and $F_N = F_{N-2} + F_{N-1}$ for all $N \geq 2$.

At the beginning of the program, the variable AA is set to N. All other variables are set to 0. At the end of the program, the variable AA must be set to the value of F_N modulo P. The values of the other variables do not matter.

For each test case, your program will be tested on 100 different values of N. These values are fixed beforehand and are the same for each test case. Your program will be accepted if it correctly calculates the value of F_N modulo P for all tested values of N.

INPUT

The first and only line contains the positive integer P, the modulus used for the MOD instruction ($3 \leq P \leq 10^9 + 7$). It is guaranteed that P is a prime number.

OUTPUT

On the first line, print an integer K ($1 \leq K \leq 10^5$) - the total number of instructions of the program.

Then, print K more lines containing the instructions as described above. Each instruction must be on a separate line.

Sample Input	Sample Output
3	5 ADD AA AA AB ADD AA AB AC MUL AB AC AD ADD AA AD AE MOD AE AF

The output is an example of a valid program.

If the initial value of AA is 1, then at the end of the program, these variables have the following values:

AA = 1, AB = 2, AC = 3, AD = 6, AE = 7, AF = 1

All other variables have a value of 0.

Note that this output will not be accepted for this input. It is only for demonstration purposes.

K. BULLETHELL

On your adventure to the Scarlet Devil Mansion, you encounter Patchouli Knowledge, who will try to stop you from advancing further.

To prevent damage to the mansion, Patchouli embedded the mansion with a space manipulation spell that temporarily removes the walls, turning the floor into an infinitely large 2D plane.

Then, she instantly casts a water manipulation spell, causing you to freeze and slide across the floor along the vector (x_t, y_t) at v units per second.

Finally, she casts q wood manipulation spells, the i-th spell can be represented by three points O_i, A_i, B_i, a start time l_i and an end time r_i.

During the i-th spell, she summons an infinitely long tree branch with a thickness of 10^{-100} along the ray O_iA_i. Then she swings it in a clockwise direction until the branch reaches the ray O_iB_i, at which point she will start swinging it in a counterclockwise direction until it reaches the ray O_iA_i. She starts swinging it back and forth at time l_i and ends this process at time r_i. After the attack, the tree branch disappears completely.

You will be hit by the i-th spell at time k if $l_i \le k \le r_i$ and your position at time k is in the swinging range of the i-th spell (it doesn't matter where the branch is currently because she swings it at light speed!). See the figures below for a visual explanation.

You will faint if you get hit by at least one spell for T consecutive seconds.

Assuming that your position was (x_0, y_0) when you were frozen, please determine if you survive Patchouli's spells.

INPUT

Each test contains multiple test cases. The first line contains the number of test cases t ($1 \le t \le 100$). The description of the test cases follows.

The first line of each test case contains 6 integers x_0, y_0, x_t, y_t, v, T ($|x_t| + |y_t| > 0$, $1 \le v, T \le 10^9$), which are the coordinates of the starting position, the movement vector, the speed at which you slide across the floor and the required time it takes for you to faint.

The next line contains one integer q ($1 \le q \le 2 \times 10^5$), the number of wood manipulation spells that Patchouli uses.

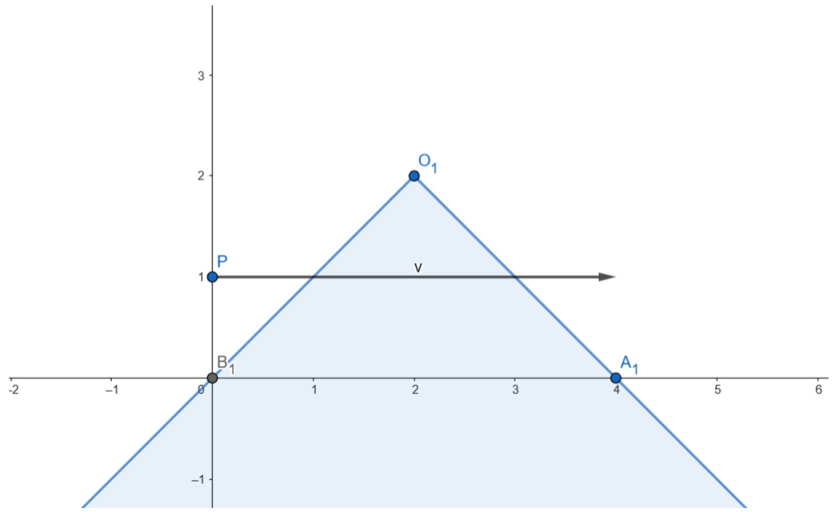
The i-th of the following q lines contains 8 integers x_{Oi}, y_{Oi}, x_{Ai}, y_{Ai}, x_{Bi}, y_{Bi}, l_i, r_i ($0 \le l_i < r_i \le 10^9$): the coordinates of O_i, A_i, B_i and the start and end time of the i-th spell. It is guaranteed that O_i, A_i, B_i are not collinear.

It is guaranteed that all coordinates (x, y) given in test cases satisfy $|x|, |y| \le 10^9$ and the sum of all q does not exceed 2×10^5.

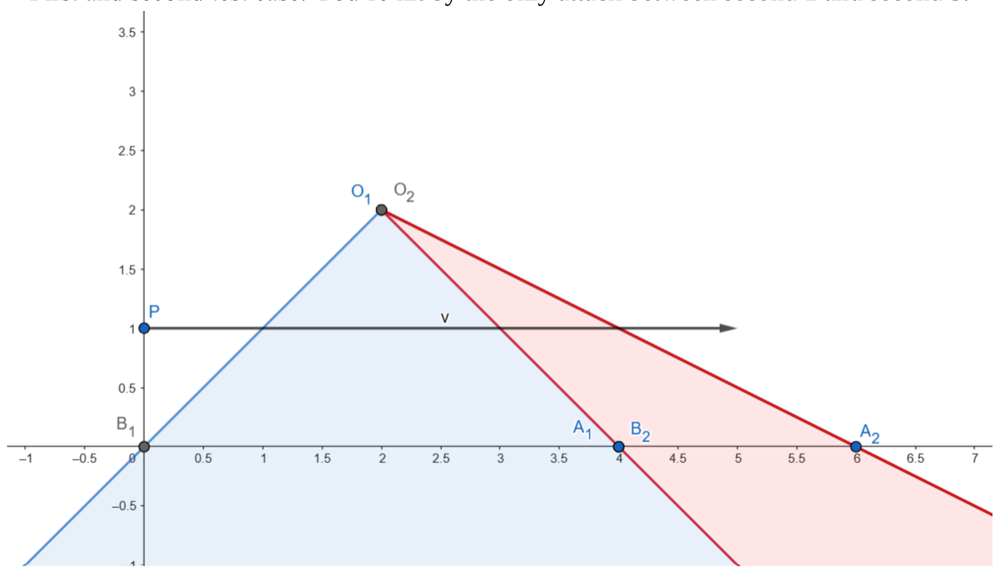
OUTPUT

For each test case, output "GIT GUD" if you survive, and "FAINTED" otherwise.

Sample Input	Sample Output
3 0 1 1 0 1 2 1 2 2 4 0 0 0 1 10 0 1 1 0 1 3 1 2 2 4 0 0 0 1 10 0 1 1 0 1 3 2 2 2 4 0 0 0 1 10 2 2 6 0 4 0 1 10	FAINTED GIT GUD FAINTED



First and second test case. You're hit by the only attack between second 1 and second 3.



Third test case. The first attack hit you between second 1 and 3, and the second attack hit you from second 3 to second 4.

L. EULER

Given 2 positive integers A and B. Calculate the sum of the Euler's totient function of the first B multiples of A. As the answer can be large, output the remainder of it when divided by 998244353.

The Euler's totient function of a positive integer n is the number of integers k such that $1 \leq k \leq n$ and $\gcd(n, k) = 1$. Here, gcd(a, b) is the greatest common divisor of a and b.

INPUT

The first and only line contains 2 integers A and B. ($1 \leq A, B \leq 5 \times 10^6$).

OUTPUT

Print one line containing the answer - the sum of the Euler's totient function for the first B multiples of A, modulo 998244353.

Sample Input	Sample Output
2 4	9
1 3	4

For the first test, we can see that:

$\varphi(2) + \varphi(4) + \varphi(6) + \varphi(8) = 1 + 2 + 2 + 4 = 9$

For the second test, we have:

$\varphi(1) + \varphi(2) + \varphi(3) = 1 + 1 + 2 = 4$